BLIND SEPARATION OF REFLECTIONS USING THE IMAGE MIXTURES RATIO

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ABSTRACT

A new method for the blind separation of linear image mixtures is presented in this paper. Such mixtures often occur, when, for example, we photograph a scene through a semireflecting medium (windshield or glass). The proposed method requires two mixtures of two scenes captured under different illumination conditions. We show that the boundary values of the ratio of the two mixtures can lead to an accurate estimation of the separation matrix. The technique is very simple, fast, and reliable, as it does not depend on iterative procedures. The method effectiveness is tested on both artificially mixed images and real images.

1. INTRODUCTION

A common problem in photography when shooting through a semireflecting medium, such as a glass or a windshield, is the projection of the reflection of the medium with the photographed scene. Consider the situation when photographing out of a room through a glass, the photos are mixtures of the world outside the room and the reflection of the room interior on the glass. The separation of the inside and the outside images using only their mixtures without *a priori* information is a very difficult task.

The problem was approached in the past using photography theory, image processing and blind separation techniques. Classic photography offers important tools (such as focus [1] and polarization [2]), that can be used in semi-automatic techniques for the separation of image mixtures. Recently, Levin *et al.*, presented a novel method [3] for separating two images from only one mixture, using typical image processing techniques. The basic idea is to find the separation that yields the minimum total amount of edges and corners on both the reconstructed images. Alternatively, the problem can be treated as a blind separation technique. Farid and Adelson in [4], under the assumption of source independence, decompose the mixing matrix using Singular Value Decomposition (SVD) and then recover each part of the decomposition. In [5], Bronstein *et al.* separates the source images by exploiting the sparseness of the sources and introducing Sparse Independent Component Analysis (Sparse ICA).

In this paper we present a new method for the separation of two source images using two linear mixture signals. The basic idea is to investigate the properties of the mixtures ratio, considering that the images are bounded signals. We discover that under some minor assumptions, the minimum and the maximum values of the mixtures ratio give a good approximation of the normalized mixing parameters. The separation using these parameters is straightforward. The method does not contain any recurrent procedure and is quick and accurate. In Section 3, the proposed technique is tested on artificially mixed images and real photos.

2. METHOD DESCRIPTION

2.1. Problem Formulation

Consider two gray scale images s_1 and s_2 of size $L \times M$. By stacking the image columns on top of each other to form a single (long) column, we can treat the images as one-dimensional signals $s_1(k)$, $s_2(k)$, with $k = 1, \cdots, N$ ($N = L \times M$). Let x_1, x_2 be two linear mixtures of s_1 and s_2:

$$x_1(k) = a_{11}s_1(k) + a_{12}s_2(k) \quad (1)$$

$$x_2(k) = a_{21}s_1(k) + a_{22}s_2(k) \quad (2)$$

where $a_{ij} > 0$ and $a_{i,1} + a_{i,2} = 1$, for $i, j \in \{1, 2\}$, $k = 1, ..., N$. The mixing matrix $\mathbf{A} = [a_{ij}]$ is assumed to be full rank, so $det(\mathbf{A}) \neq 0$. We may view the mixture images as linear superpositions of an image scene (s_1) over a reflection scene (s_2). However, this may not be the only possible interpretation of Eqs. (1) and (2).

Each pixel $s_1(k)$ or $s_2(k)$ is an integer bounded between 0 and $Q = 2^q - 1$, where $q > 0$ is the bit-depth (typically $q = 8$ and $Q = 255$). Since the scale of the source signals

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is unobservable, we may define the scaled source signals

$$y_1 \triangleq a_{11}s_1, \qquad y_2 \triangleq a_{12}s_2,$$

and use them to rewrite equations (1), (2) as follows:

$$x_1(k) = y_1(k) + y_2(k) \tag{3}$$

$$x_2(k) = b_1y_1(k) + b_2y_2(k) \tag{4}$$

where $b_1 = a_{21}/a_{11}$ and $b_2 = a_{22}/a_{12}$. Then the ratio of the mixture signals is:

$$r_x(k) \triangleq \frac{x_2(k)}{x_1(k)} = \frac{b_1 + b_2r_y(k)}{1 + r_y(k)} \tag{5}$$

where $r_y(k) = y_2(k)/y_1(k)$. Since the mixtures are known the ratio $r_x(k)$ can be computed through pixel by pixel division: $x_2(k)/x_1(k)$, $k = 1, \cdots, N$. Note that, according to (5), r_x is a monotonic function of r_y. Indeed, the derivative

$$\frac{\partial r_x}{\partial r_y} = \frac{b_2 - b_1}{(1 + r_y(k))^2}$$

has constant sign equal to $+1$ or -1 depending on the relative sizes of b_1 and b_2. If $b_2 > b_1$ then r_x is a monotonically increasing function of r_y, while for $b_2 < b_1$, r_x is a monotonically decreasing function of r_y. The case $b_1 = b_2$ is not considered since it would imply $a_{21}/a_{11} = a_{22}/a_{12}$ and therefore $a_{11}a_{22} - a_{21}a_{12} = 0$ which would violate the assumption $det(\mathbf{A}) \neq 0$. Since

$$0 \leq r_y \leq \infty \tag{6}$$

the limits of r_x are:

$$r_x\big|_{r_y=0} = b_1, \qquad r_x\big|_{r_y=\infty} = b_2 \tag{7}$$

It follows that

$$\min(b_1, b_2) \leq r_x(k) \leq \max(b_1, b_2) \tag{8}$$

with equality when either $r_y = 0$ or $r_y = \infty$. Assume that the extreme values $r_y(k) = 0$ and $r_y(k) = \infty$ are reached at some pixels $k = k_1$ and $k = k_2$. Then, we can estimate b_1 and b_2 (not necessarily in that order) by $\widehat{b_1} = \min_k\{r_x(k)\}$ and $\widehat{b_2} = \max_k\{r_x(k)\}$. Indeed, there are two cases:

case 1: if $b_1 < b_2$, then $\widehat{b_1} = r_x(k_1) = r_x\big|_{r_y=0} = b_1$ and $\widehat{b_2} = r_x(k_2) = r_x\big|_{r_y=\infty} = b_2$;

case 2: if $b_1 > b_2$, then $\widehat{b_1} = r_x(k_2) = r_x\big|_{r_y=\infty} = b_2$, and $\widehat{b_2} = r_x(k_1) = r_x\big|_{r_y=0} = b_1$.

Once the mixing parameters are retrieved, the sources y_1 and y_2 can be estimated by inverting the mixing matrix:

$$\begin{bmatrix} \widehat{y_1}(k) \\ \widehat{y_2}(k) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \widehat{b_1} & \widehat{b_2} \end{bmatrix}^{-1} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}. \tag{9}$$

Relating to the two cases discussed above, we have $\widehat{y_1} = y_1$, $\widehat{y_2} = y_2$ (case 1), and $\widehat{y_1} = y_2$, $\widehat{y_2} = y_1$ (case 2). In both cases we have perfect reconstruction of the images y_1, y_2, although their order may be confused. Of course, our ultimate goal is to obtain the original images s_1 and s_2 which are related to y_1 and y_2 through an unknown scaling. Although the true scaling factors are impossible to retrieve, we may alleviate the problem by assuming that the histograms of s_1, s_2, range from 0 to 255 and thus we may scale $\widehat{y_1}$ and $\widehat{y_2}$ to fit these limits. Based on the above discussion, the following blind separation method is proposed:

Step 1. Calculate the mixture ratio $r_x(k) = x_2(k)/x_1(k)$;

Step 2. Find the minimum and the maximum values of r_x. Let $\widehat{b_1} = \min(r_x)$ and $\widehat{b_2} = \max(r_x)$;

Step 3. Estimate the source images through Eq. (9);

Step 4. Rescale the sources in the luminance range $[0, 255]$.

2.2. Estimation accuracy

From Eq. (7), it is obvious that the accuracy of the estimation depends on how well the conditions $r_y = 0$ or $r_y = \infty$ are met, or approximated. Perfect estimation implies that there exist at least two indices k_1, k_2 such that

$$\begin{array}{c} s_1(k_1) = 0 \text{ and } s_2(k_1) \neq 0 \rightarrow r_y(k_1) = \infty \\ s_1(k_2) \neq 0 \text{ and } s_2(k_2) = 0 \rightarrow r_y(k_2) = 0 \end{array} \tag{10}$$

namely, there must be at least two pixel coordinates k_1 and k_2, where s_1 is black while s_2 is not, and vice-versa. Since the sources are unknown, it is impossible to verify the validity of these conditions. However, it must be noted that this assumption does not heavily limit the method. Even if the source images are not very "dark", in most cases there exist two pixels k'_1, k'_2, such that $r_y(k'_1)$ is close to zero and $r_y(k'_2)$ is large. To see how this affects our estimation, let

$$\alpha = \min_k(r_y(k)) \text{ and } \beta = \max_k(r_y(k))$$

Then

$$\widehat{b_1} = \frac{b_1 + \alpha b_2}{1 + \alpha} = b_1 + \frac{\alpha}{1 + \alpha}(b_2 - b_1)$$

$$\delta b_1 = \widehat{b_1} - b_1 = \frac{\alpha}{1 + \alpha}(b_2 - b_1) \tag{11}$$

and similarly,

$$\delta b_2 = \widehat{b_2} - b_2 = \frac{1}{1 + \beta}(b_1 - b_2) \tag{12}$$

So, for example, if $\alpha = 0.1$ and $\beta = 10$, then the estimation errors $|\delta b_1| = |\delta b_2| = 0.09|b_1 - b_2|$, are a small fraction of the distance $|b_1 - b_2|$ between the mixing parameters.

The performance of the method can be further improved by preprocessing the signals using a linear transformation (for instance, Fourier Transform) since, in the transform domain, the limits of the source ratio r_y are usually much closer to the absolute limits, zero and infinity.

3. RESULTS

The method was tested with artificially mixed images and real mixture scenes photographed through a semireflective glass.

3.1. Artificial Data

We present two experiments with mixtures of well known images obtained using different mixing parameters. In the first experiment the mixing matrix is far from singular while, in the second one, the mixing parameters for the two mixtures are almost identical and therefore, the mixing matrix is almost singular.

Fig. 1 (a)(b) shows the original images "F16" and "Sailboat" used in the first experiment. These images were mixed into the ones shown in Fig. 1 (c)(d). The following mixing matrix was used

$$\mathbf{A} = \begin{bmatrix} 0.6102 & 0.3898 \\ 0.3223 & 0.6777 \end{bmatrix} \tag{13}$$

The true values of b_1 and b_2, are 0.5282 and 1.7386, respectively, while the estimated values obtained by our method are $\widehat{b_1} = 1.7386, \widehat{b_2} = 0.5934$. It can be witnessed that the reconstructed images (Fig. 1 (e)(f)) are very similar to the original ones even though the "F16" image does not contain large black areas. The good quality of the separation is also supported by the fact that the demixing operator is very close to the identity matrix (with permuted columns):

$$\begin{bmatrix} 1 & 1 \\ \widehat{b_1} & \widehat{b_2} \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ b_1 & b_2 \end{bmatrix} = \begin{bmatrix} -0.0569 & 1.0000 \\ 1.0569 & 0.0000 \end{bmatrix} \tag{14}$$

The source images "Lenna" and "Cameraman" that were used in the second experiment are shown in Fig. 2 (a)(b). For this experiment the mixing matrix

$$\mathbf{A} = \begin{bmatrix} 0.6102 & 0.3898 \\ 0.6103 & 0.3899 \end{bmatrix} \tag{15}$$

is almost singular: indeed, the mixing parameters for the two mixtures differ at the fourth decimal point. Naturally, the two mixtures look identical (see Fig. 2 (c)(d)). Nevertheless, the reconstructed images (Fig. 2 (e)(f)) are again very similar to the original ones. Again the demixing operator is close to the identity matrix with permutated columns:

$$\begin{bmatrix} 1 & 1 \\ \widehat{b_1} & \widehat{b_2} \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ b_1 & b_2 \end{bmatrix} = \begin{bmatrix} -0.0252 & 1.0437 \\ 1.0252 & -0.0437 \end{bmatrix} \tag{16}$$

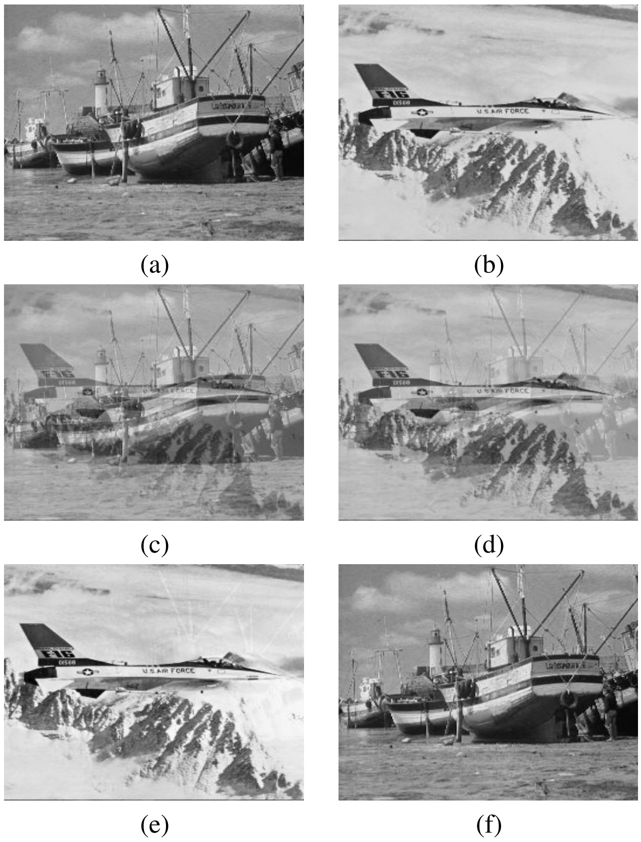

Fig. 1. (a)(b) Source images used in the first experiment; (c)(d) the mixed signals; (e)(f) the reconstructed images.

3.2. Real Images

For the creation of real data we photographed a painting framed behind glass. The pictures were shot at a certain angle so that an image from the surrounding environment (curtain) was reflected on the glass and mixed with the image of the painting. We took two photographs under two different illumination conditions (Fig. 3 (a)(b)). We processed only the image part where the reflectance is present (Fig. 3 (c)(d)). The results of the blind separation method are depicted in Figure 3 (e)(f). Although it is not possible to numerically evaluate the performance of the method (since the true mixing parameters are unknown) we can see, by simple inspection, that the quality of the separation is good.

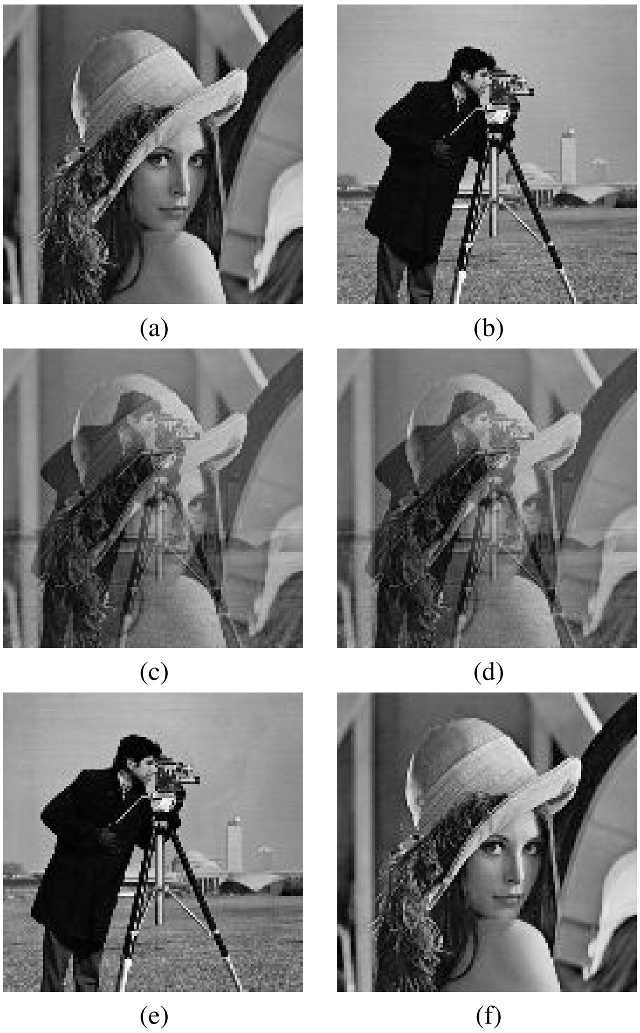

Fig. 2. (a)(b) Original images "Lenna" and "Cameraman" of the second experiment; (c)(d) mixtures of the two originals with almost identical mixing parameters; (e)(f) the reconstructed images.

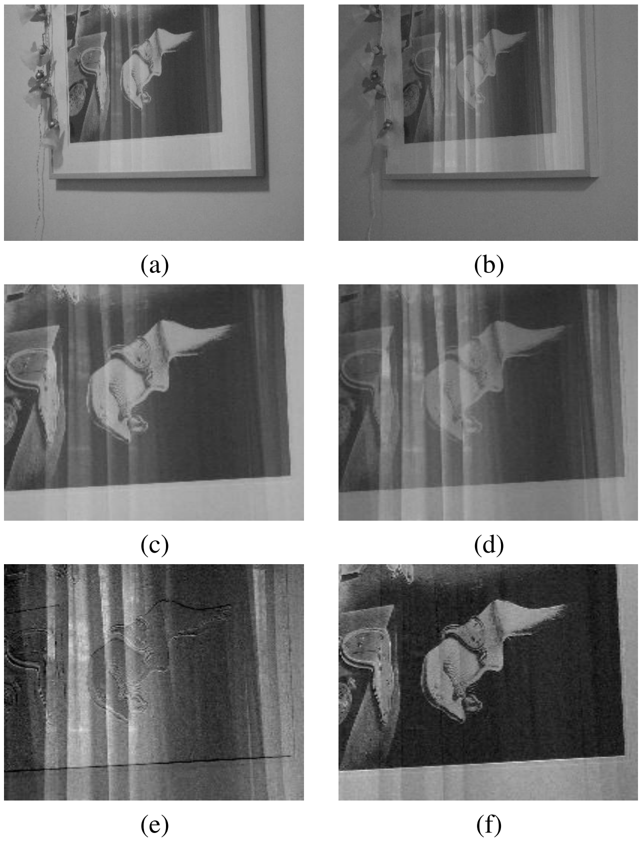

Fig. 3. (a)(b) Two photos of a framed picture under different illumination conditions, with a curtain reflecting on the glass; (c)(d) the image parts containing only the reflection-contaminated area; (e)(f) the separated images.

4. CONCLUSION

In this paper we presented a novel method for the separation of reflections from two images taken through a semireflective medium. The method explores the properties of the ratio of the two mixtures and the boundary values can lead to the estimation of the mixing matrix. The proposed algorithm is very simple, fast and has very good estimation accuracy. Results over real and artificial mixtures show the efficiency of the method under various conditions.

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