Blind Deconvolution of SISO Systems with Binary Source based on Recursive Channel Shortening

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Abstract. We treat the problem of Blind Deconvolution of Single Input - Single Output (SISO) systems with real or complex binary sources. We explicate the basic mathematical idea by focusing on the noiseless case. Our approach leads to a recursive channel shortening algorithm based on simple data gouping. The channel shortening process eventually results in an instantaneous binary system with trivial solution. The method is both deterministic and very fast. It does not involve any iterative optimization or stochastic approximation procedure. It does however, require sufficiently large datasets in order to meet the source richness condition.

1 Introduction

Binary signals received a lot of attention in the last decades, due to their application in digital/wireless communications. A typical problem in this area is the blind separation of multiple signals arriving at an antenna array when the sources are BPSK [1]. In [2], van der Veen investigated the problem of instantaneous Blind Source Separation (BSS) with binary sources, when the mixing operator is complex. The proposed solution is based on generalized eigenvalue decomposition and it is non-iterative. Diamantaras *et al.* in [3] found an analytical method to decompose mixtures of binary sources using only one observation. Similarly, Li *et al.* proposed a similar method, [4], where solvability of the problem was presented in details.

The Blind Deconvolution of a $n_1 \times n_2$ MIMO system consists on the estimation of the n_1 input signals given only the n_2 output signals, while the convolutive system is unknown. The solution was given through the eigen-decomposition of the output correlation matrix in [5]. In [6] the MIMO problem was reduced into multiple SIMO problems by multiplying each observation by its conjugate cyclic frequency. Ma *et al.* [7] proposed a method using the generalized eigenvalue decomposition of a matrix pencil formed by output auto-correlation matrices at different time-lags. A class of algorithms exploiting the statistical independence of the sources was presented in [8], by Yellin *et al.*. In [9], Tugnait proposed a cumulant maximization-based approach that decomposes the mixtures into sets of independent signal components at each sensor.

In this paper we investigate two similar problems: (a) Blind Deconvolution of real SISO systems with a real binary source, and (b) Blind Deconvolution of complex SISO systems with a complex source having binary real and imaginary parts. Both problems are studied in their ideal, noiseless case, because ths approach best exposes the underlying mathematical concepts. The introduction of noise would require a straightforward modification of the proposed algorithm. Necessary assumptions for the problem solvability are presented. In general, these assumptions require large datasets so that the source data set is rich enough in binary combinations. The proposed scheme recursively eliminates the channel parameters, i.e. it performs channel shortening. Once this iterative process concludes, the resulting system is a memoryless one. From that, the binary source estimation is straightforward. The proposed scheme is further developed in order to be applied in the case of complex source with binary parts and complex mixing operators. The paper concludes with results in both cases.

2 Blind Deconvolution of a SISO system: real Binary Source

Let us observe the output of a real, noiseless SISO system

$$x(k) = \sum_{l=0}^{L} a(l)s(k-l), \quad k = 1, \cdots, K \tag{1}$$

or

$$x(k) = \mathbf{a}^T \mathbf{s}(k), \quad k = 1, \cdots, K \tag{2}$$

where $\mathbf{a}$ is the real, $(L+1)$-dimensional vector, x is the system output, and

$$\mathbf{s}(k) = [s(k), s(k-1), ..., s(k-L)]^T,$$

is the binary source $s(k) \in \{-1, +1\}$. Since the binary vector $\mathbf{s}$ has length $L+1$, it can take 2^{L+1} distinct values. Consequently, $x(k) \in \mathcal{X}$ can take at most 2^{L+1} values, i.e. $|\mathcal{X}| \leq 2^{L+1}$. We make the following assumption:

Assumption 1 *Every possible value of the output $x(k)$ corresponds to a unique source vector $\mathbf{s}(k)$, therefore the cardinality of $\mathcal{X}$ is exactly 2^{L+1}.*

This simply means that no pair of distinct sources will produce the same output. For the similar problem of blindly separating multiple binary sources $s_i(k)$, from a single linear mixture $x(k) = \sum_i a(i)s_i(k)$, Li *et al.* in [4] concluded that the following condition ensures solvability:

$$c_0 a(i_0) + ... + c_N a(i_N) \neq 0, \tag{3}$$

for any coefficients $c_k \in \{-1, +1\}$, and any subset $\{i_0, ..., i_N\}$ $(N < L)$ of $\{0, 1, ..., L\}$.

Take any time instant k_0, and let $\mathbf{s}(k_0) = [c_0, c_1, ..., c_L]$, $c_i \in \{-1, +1\}$, be the source vector yielding the output $x(k_0) = r$. The successor observation at time instant $k_0 + 1$, can assume only two possible values, $r^s_{(1)}$ or $r^s_{(2)}$, depending on the corresponding source vector which can take one of the following two values $\mathbf{s}_{(1)}(k_0+1) = [+1, c_0, c_1, ..., c_{L-1}]$ or $\mathbf{s}_{(2)}(k_0+1) = [-1, c_0, c_1, ..., c_L]$. In our method it is essential that both pairs of consecutive values $[r, r^s_{(1)}]$ and $[r, r^s_{(2)}]$, will appear, at least once, in the output sequence $x(k)$, $k = 1, \cdots, K$. This is stated in the following assumption:

Assumption 2 *For any $r \in \mathcal{X}$, there are at least two indices $k_0, k_1 \in \{1, 2, \cdots, K\}$ such that $x(k_0) = r$, $x(k_0+1) = r^s_{(1)}$ and $x(k_1) = r$, $x(k_1 + 1) = r^s_{(2)}$.*

This assumption, of course, requires that the dataset is large enough. The successors $r^s_{(1)}$, $r^s_{(2)}$, of r can be found by simple observation of the output data set. Once this is done, it is straightforward to estimate $|a(0)|$ as follows:

$$\begin{aligned} \left| r^s_{(1)} - r^s_{(2)} \right| &= \left| \mathbf{a}^T \mathbf{s}_{(1)}(k_0+1) - \mathbf{a}^T \mathbf{s}_{(2)}(k_0+1) \right| \\ &= |\, a(0)(+1 - (-1)) \,| \\ &= 2|a(0)| \end{aligned} \tag{4}$$

Moreover, the sum $\rho(r)$ of the successors of r is

$$\rho(r) = r^s_{(1)} + r^s_{(2)} = \mathbf{a}^T \mathbf{s}_{(1)}(k_0+1) + \mathbf{a}^T \mathbf{s}_{(2)}(k_0+1)$$

$$= a(0)(+1 + (-1)) + 2\sum_{i=1}^{L} a(i)c_i$$

$$= 2\sum_{i=1}^{L} a(i)c_i \tag{5}$$

Estimating $\rho(r)$ for every $r \in \mathcal{X}$ can lead to a new SISO system with a shortened channel. Indeed, substituting every observation $x(k) = r$ with $\rho(r)/2$, we obtain:

$$x^{(2)}(k) = \rho(r)/2 = \sum_{l=1}^{L} a(l)s(k-l) \tag{6}$$

It is clear that the new SISO system in Eq. 6 has the same taps as the original one in Eq. 2 except for the lack of $a(0)$. Of course the length of the new system is L, i.e. one less than the initial length $L+1$. The above transformation can be recursively applied L times until the system is reduced into:

$$x^{(L+1)}(k) = a(L)s(k-L) = \pm a(L) \tag{7}$$

Since, system (7) is non-convolutive, the source estimation is a straightforward process. Notice that, at any time instant k, the output $x^{(L+1)}(k)$ will assume one of two values $+a(L)$ or $-a(L)$. So we can easily estimate the absolute value of the last filter tap as $\hat{a}(L) = |\,x^{(L+1)}(k)\,| = |\,a(L)\,|$, (any k) and from that we can estimate the source by:

$$\hat{s}(k-L) = x^{(L+1)}(k)/\hat{a}(L) = \sigma s(k-L) \tag{8}$$

where $\sigma = \pm 1$. In this process, of course, we lose the sign information but it is well known that the source sign is unobservable.

3 SISO Blind Deconvolution: Complex Binary Source

In the sequel we shall adopt the following notation convention: for any complex number c, c_R and c_I will denote the real and the imaginary parts of c respectively.

Now let us reconsider the SISO system of Eq. (2) where $\mathbf{a}$ is a complex, $(L+1)$-tap vector. The source $s(k)$ is also complex but with binary real and imaginiary components: $s_R(k), s_I(k) \in \{1, -1\}$, i.e. $s(k) \in \mathcal{B} = \{1 + j, 1 - j, -1 + j, -1 - j\}$. As in the real case, the complex source vector $\mathbf{s}(k)$ has length $L+1$. Clearly, $\mathbf{s}(k)$ can take 4^{L+1} distinct values, and $x(k) \in \mathcal{X}_c$ can take at the most 4^{L+1} distinct values. Similarly to section 2 we assume that

Assumption 3 *Every possible value of the output $x(k)$ corresponds to a unique source vector $\mathbf{s}(k)$, therefore $|\mathcal{X}_c| = 4^{L+1}$.*

As in the real case, for any time instant k_0, the output $x(k_0) = r$, comes from a unique source vector $\mathbf{s}(k_0) = [c_0, c_1, ..., c_{L-1}, c_L]^T$, $c_i \in \mathcal{B}$. The successor vector, at time $k_0 + 1$, can now take four possible values, $r^s_{(1)}$, $r^s_{(2)}$, $r^s_{(3)}$, and $r^s_{(4)}$ as follows:

$$\begin{aligned}
\mathbf{s}_{(1)}(k_0+1) &= \left[1+j, c_1, ..., c_{L-1}, c_L\right]^T, \\
\mathbf{s}_{(2)}(k_0+1) &= \left[1-j, c_1, ..., c_{L-1}, c_L\right]^T, \\
\mathbf{s}_{(3)}(k_0+1) &= \left[-1+j, c_1, ..., c_{L-1}, c_L\right]^T, \\
\mathbf{s}_{(4)}(k_0+1) &= \left[-1-j, c_1, ..., c_{L-1}, c_L\right]^T.
\end{aligned} \tag{9}$$

Again, it is essential that all the pairs of output values $[r, r^s_{(i)}], i = 1,2,3,4$, will appear, at least once, in the output sequence $x(k)$, $k = 1, \cdots, K$:

Assumption 4 *For any $r \in \mathcal{X}_c$, there are at least four indices $k_0, k_1, k_2, k_3 \in \{1, 2, \cdots, K\}$ such that $x(k_0) = r$, $x(k_0+1) = r^s_{(1)}$, $x(k_1) = r$, $x(k_1 + 1) = r^s_{(2)}$, $x(k_2) = r$, $x(k_2+1) = r^s_{(3)}$, and $x(k_3) = r$, $x(k_3+1) = r^s_{(4)}$.*

Once we find the successors $r^s_{(i)}$, of a specific observation value $x(k) = r$, we compute $\rho(r)$:

$$\begin{aligned}
\rho(r) &= \sum_{i=1}^{4} r^s_{(i)} = \sum_{i=1}^{4} \mathbf{a}^T \mathbf{s}_{(i)}(k+1) \\
&= [(1+j) + (1-j) + (-1+j) + (-1-j)]a(0) + 4\sum_{l=1}^{L} a(l)s(k-l) \\
&= 4\sum_{l=1}^{L} a(l)s(k-l)
\end{aligned} \tag{10}$$

Thus substituting $x(k) = r$ by $\rho(r)/4$, $\forall k$, we obtain a shortened system:

$$x^{(2)}(k) = \rho(r)/4 = \sum_{l=1}^{L} a(l)s(k-l). \tag{11}$$

As in the real case, L repetitions lead to a memoryless system:

$$x^{(L+1)}(k) = a(L)s(k-L) \tag{12}$$

Now the source can be estimated only up to a multiplier $\lambda \in \{\pm 1, \pm j\}$. Indeed, λ is unobservable since

$$\begin{aligned} x^{(L+1)}(k) &= a(L)s(k-L) \\ &= (-1 \cdot a(L))(-1 \cdot s(k-L)) \\ &= (-j \cdot a(L))(j \cdot s(k-L)) \\ &= (j \cdot a(L))(-j \cdot s(k-L)) \end{aligned} \tag{13}$$

Using the memoryless system (12) we can estimate the source taking again a two step approach: First, we introduce the estimate $\hat{a}(L)$ and we randomly select any time instant k_0, assuming that $x(k_0) = a(L)s(k_0 - L) = \hat{a}(L)(1+j)$. We call $\lambda^{-1} = s(k_0 - L)/(1+j)$ and we note that $\lambda^{-1} \in \{\pm 1, \pm j\}$. So $\hat{a}(L) = x^{(L+1)}(k_0)/(1+j) = \lambda^{-1}a(L)$. Second, we estimate $s(k-L)$ by

$$\hat{s}(k-L) = x^{(L+1)}(k)/\hat{a}(L) = \lambda s(k-L). \tag{14}$$

4 Examples

Example 1. In this experiment we created a source dataset of 10,000 random binary numbers. This was convolved with a real valued filter of length 10 with coefficients randomly chosen in the interval $[-1, 1]$. Table 1 presents the true filter coefficients and their estimated absolute values. Furtermore, in this example, $\hat{s}$ is a perfect estimate of the true source, except for the sign: $\hat{s}(k) = -s(k)$, $\forall k$.

	True a	Estimated $\lvert a \rvert$		True a	Estimated $\lvert a \rvert$
1	0.9235	0.9235	6	-0.1067	0.1067
2	-0.9398	0.9398	7	-0.6507	0.6507
3	0.9075	0.9075	8	0.6705	0.6705
4	0.4288	0.4288	9	0.9402	0.9402
5	0.2931	0.2931	10	-0.7301	0.7301

Table 1. The true filter coefficients and their estimated absolute values.

Example 2. In this example we used a source dataset of 20,000 complex binary samples. The following randomly generated complex filter of length three was used: $[a(0), a(1), a(2)]^T = [0.9003 - 0.0280j, -0.5377 + 0.7826j, 0.2137 - 0.5242j]^T$. Our estimated source was a perfect estimate of the true one except for the multiplier $\lambda = -j$.

5 Discussion and Conclusion

A novel blind method for deconvolving SISO systems with binary real or complex sources was presented in this paper. The method is based on the recursive shortening of the channel leading eventually in a linear memoryless system with trivial solution. In this work we only study the noiseless situation because we want to emphasize on the mathematical development of our approach. In this context, both examples presented above are simple verifications of the method. However, noise can be handled, as well, by a simple modification of the algorithm. For example, in a real SISO system with noise, we observe $y(k) = x(k) + e(k)$ instead of $x(k)$, where $e(k)$ is the noise component. In this case, the set $\mathcal{X}$ which contains the possible values of $x(k)$ has to be estimated using some clustering technique. One also needs a classification rule which will group the observations $y(k)$ into the proper values $x(k) = r$ of $\mathcal{X}$. Once this is achieved, the method can proceed as presented. The effects of noise in the algorithm performance will be studied in another contribution.

References

1. Anand, K., Mathew, G., Reddy, V.U.: Blind separation of multiple co-channel bpsk signals arriving at an antenna array. IEEE Signal Processing Letters **2** (1995) 176–178
2. van der Veen, A.J.: Analytical method for blind binary signal separation. IEEE Trans. on Signal Processing **45** (1997) 1078–1082
3. Diamantaras, K.I., Chassioti, E.: Blind separation of n binary sources from on observation: A deterministic approach. In: Proc. Second Int. Workshop on ICA and BSS, Helsinki, Finland (2000) 93–98
4. Li, Y., Cichocki, A., Zhang, L.: Blind separation and extraction of binary sources. IEICI Trans. Fundamentals **E86-A** (2003) 580–589
5. Gurelli, M.I., Nikias, C.L.: Evam: An eigenvector-based algorithm for multichannel blind deconvolution of input colored signals. IEEE Trans. Signal Processing **43** (1995) 134–149
6. Chevreuil, A., Loubaton, P.: Mimo blind second-order equalization method and conjugate cyclostationarity. IEEE Trans. Signal Processing **47** (1999) 572–578
7. Ma, C.T., Ding, Z., Yau, S.F.: A two-stage algorithm for mimo blind deconvolution of nonstationary colored signals. IEEE Trans. Signal Processing **48** (2000) 1187–1192
8. Yellin, D., Weinstein, E.: Criteria for multichannel signal separation. IEEE Trans. on Signal Processing **42** (1994) 2158–2168
9. Tugnait, J.K.: On blind separation of convolutive mixtures of independent linear signals in unknown additive noise. IEEE Trans. Signal Processeing **46** (1998) 3117–3123