ON BLIND IDENTIFIABILITY OF FIR-MIMO SYSTEMS WITH CYCLOSTATIONARY INPUTS USING SECOND ORDER STATISTICS

Ivan Bradaric, Athina P. Petropulu and Konstantinos I. Diamantaras*

Electrical & Computer Eng. Dept., Drexel University, Philadelphia, PA 19104
{athina,ivan}@cbis.ece.drexel.edu

*Department of Informatics, Technological Education Institute, Sindos, GR-54101, Greece
kdiamant@it.teithe.gr

ABSTRACT

We consider a general $n \times n$ MIMO system excited by unobservable inputs that are spatially independent, cyclostationary with unknown statistics. We provide a set of conditions under which the system is uniquely identifiable based on second-order frequency-domain correlations of the system output. Such a MIMO problem appears in many applications, such as multi-user communications and separation of competing speakers.

1. INTRODUCTION

The goal of blind $n \times n$ Multiple-input multiple-output (MIMO) system identification is to identify an unknown convolutive system, driven by n unobservable inputs, based on the system n outputs. MIMO problems are of great importance in many applications, such as communications, biomedical engineering, seismology, etc.. In this paper we are concerned with a special case of inputs that are *cyclostationary* with *unknown statistics*. A MIMO estimation problem for such case would apply to blind channel estimation in multi-user communications applications, and also to the estimation of the acoustic channel to be used for separating n competing speech signals based on n microphone recordings.

It is well known that, in its most general form, the problem is ill posed and admits infinite number of solutions. A commonly used assumption in blind MIMO estimation problems with unknown input statistics is that the outputs are cross-wise mixtures of the inputs (unity auto-channels). The stationary input case has been approached using either second-order statistics ([7], [5]) or higher-order statistics of the system outputs ([12], [10], [6]), and the non-stationary input case has been studied in [8] via second-order statistics. However, even under the unity auto-channels constraint, the obtained system matrix contains some form of ambiguity, i.e., in [8], [5] it was a frequency dependent permutation ambiguity, in [7] it was appearance of the "tricky solution" (note that this paper addressed only 2x2 case), in [10] it was constant permutation matrix. The only work that has shown the existence of a unique solution for the stationary input case is that of [6], and there the solution was based on fourth-order statistics of the system output. We should note that no algorithm has been proposed in [6] for reaching the unique solution.

In this paper we exploit the cyclostationarity of the inputs and show that a cross-wise $n \times n$ convolutive mixture is uniquely identifiable based on second order statistics of the system output. We should note here that some of the above mentioned papers do apply to the case of cyclostationary inputs ([10]), however, no particular way has been proposed to exploit cyclostationarity for improving identifiability.

The proof of identifiability given in this paper involves a sequence of steps that basically recover the system matrix. However, at this point, when estimates rather than true statistics are employed in these steps, the estimation is rather sensitive. Thus, further work is needed if one would wish to turn this identifiability result to a system estimation method.

This work was supported by NSF under grant MIP-9553227.

2. PROBLEM FORMULATION

Let $\mathbf{s}(k) = [s_1(k) \cdots s_n(k)]^T$ be a vector of n statistically independent zero mean cyclostationary sources, $\mathbf{H}(l)$ the impulse response matrix with elements $\{h_{ij}(l)\}$, and $\mathbf{x}(k) = [x_1(k) \cdots x_n(k)]^T$ the vector of observations. The system output equals:

$$\mathbf{x}(k) = \sum_{l=0}^{L-1} \mathbf{H}(l)\mathbf{s}(k-l) \quad (1)$$

where L is the length of the longest $h_{ij}(k)$, and $s_i(k)$ is the i-th source signal. By taking the discrete-time Fourier transform of Eq. (1), we obtain:

$$\mathbf{x}(\omega) = \mathbf{H}(\omega)\mathbf{s}(\omega) \quad (2)$$

where $\mathbf{x}(\omega)$, $\mathbf{H}(\omega)$ and $\mathbf{s}(\omega)$ are the discrete-time Fourier transform of $\mathbf{x}(k)$, $\mathbf{H}(k)$ and $\mathbf{s}(k)$, respectively and $\omega \in [0, 2\pi)$. From the last expression it is obvious that at each ω, estimation of $\mathbf{H}(\omega)$ can be viewed as an instantaneous MIMO problem. There are many works that address the latter problem, for example [11], [2], [1], [3]. In this paper we are interested in blind identification based on second-order statistics only and will refer to the works presented in [11] and [1].

Let us at first define the covariance matrix of the stochastic process $\mathbf{s}(\omega)$ as:

$$\mathbf{R}_s(\omega_1, \omega_2) \triangleq E\{\mathbf{s}(\omega_1)\mathbf{s}(\omega_2)^H\} \quad (3)$$

and introduce the following assumptions:

(A1) The inputs $\{s_i(k)\}$, $i = 1, \cdots, n$ are pairwise uncorrelated, non-i.i.d. with unknown statistics, and cyclostationary.

(A2) There exists $\alpha \neq 0$ such that:

$$E\{s_i(\omega)s_i(\omega+\alpha)^H\} \neq E\{s_j(\omega)s_j(\omega+\alpha)^H\}\ for\ i \neq j \quad (4)$$

for all $\omega \in [0, 2\pi)$.

(A3) $rank[\mathbf{H}(\omega)] = n$ for all $\omega \in [0, 2\pi)$.

(A4) Diagonal entries of channel matrix $\mathbf{H}(\omega)$ are equal to 1, that is, $H_{ii}(\omega) = 1, i = 1, \cdots, n$.

(A5) The mixing channels $h_{ij}(k)$ are FIR filters that have no common zeros and are in general complex.

(A6) The channels $h_{ij}(n)$ and $h_{ik}(n),\ i \neq j,\ i \neq k$, have no zeros in conjugate reciprocal pairs.

We should note here that assumption (A2) cannot be met in the case of stationary inputs, since $E\{s_i(\omega)s_i(\omega+\alpha)^H\}$, $i = 1, \cdots, n$ vanishes for any $\alpha \neq 0$. The need for assumptions (A5) and (A6) will become apparent later when we try to resolve the column permutation ambiguity associated with the system identification.

Proposition 1: *Under the assumptions (A1)-(A3), the following function of the system matrix can be obtained:*

$$\hat{\mathbf{H}}(\omega) = \mathbf{H}(\omega)\mathbf{P}(\omega)\mathbf{\Lambda}(\omega) \quad (5)$$

where $\mathbf{P}(\omega)$ is a column permutation matrix, and $\mathbf{\Lambda}(\omega)$ is a complex diagonal matrix.

Proof: Since the input sequences $\{s_i(k)\}$ are assumed uncorrelated, $\mathbf{R}_s(\omega_1, \omega_2)$ is diagonal matrix, complex in general, except for $\omega_1 = \omega_2$ when it is real. For a fixed ω, based on (1) and assumptions (A2) and (A3), *Theorem 2* of [11] asserts that $\mathbf{H}(\omega)$ can be reconstructed up to the permutation and scaling ambiguity, i.e., one can obtain:

$$\hat{\mathbf{H}}(\omega) = \mathbf{H}(\omega)\mathbf{P}(\omega)\mathbf{\Lambda}(\omega) \quad (6)$$

Q.E.D.

Unfortunately, both $\mathbf{P}(\omega)$ and $\mathbf{\Lambda}(\omega)$ depend on frequency, thus combining $\hat{\mathbf{H}}(\omega)$ for all frequencies is a serious problem. The effect of $\mathbf{\Lambda}(\omega)$ can be avoided by invoking assumption (A4). In that case, in the absence of the permutation ambiguity, one would obtain:

$$\tilde{\mathbf{H}}(\omega) \triangleq \hat{\mathbf{H}}(\omega)\{diag\hat{\mathbf{H}}(\omega)\}^{-1} \quad (7)$$

which would be equal to $\mathbf{H}(\omega)$. However, due to the unknown permutation matrix $\mathbf{P}(\omega)$, $\tilde{\mathbf{H}}(\omega) \neq \mathbf{H}(\omega)$. For example, let us consider the 3×3 case, in where:

$$\mathbf{H}(\omega) = \begin{bmatrix} 1 & H_{12}(\omega) & H_{13}(\omega) \\ H_{21}(\omega) & 1 & H_{23}(\omega) \\ H_{31}(\omega) & H_{32}(\omega) & 1 \end{bmatrix} \quad (8)$$

and let us assume that for some fixed ω the permutation matrix equals:

$$\mathbf{P}(\omega) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (9)$$

The above form implies that the effect of postmultiplying any matrix by $\mathbf{P}(\omega)$ would be the interchange of the first and second columns of the matrix. In that case, $\tilde{\mathbf{H}}(\omega)$ would equal:

$$\tilde{\mathbf{H}}(\omega) = \begin{bmatrix} 1 & 1/H_{21}(\omega) & H_{13}(\omega) \\ 1/H_{12}(\omega) & 1 & H_{23}(\omega) \\ H_{31}(\omega)/H_{12}(\omega) & H_{32}(\omega)/H_{21}(\omega) & 1 \end{bmatrix} \quad (10)$$

There are five more possibilities for $\tilde{\mathbf{H}}(\omega)$, depending on the form of $\mathbf{P}(\omega)$. By examining all these possibilities one can see that the only case where the coefficients do not contain ratios of mixing filters is when $\mathbf{P}(\omega) = \mathbf{I}$. In [10], where matrix $\mathbf{P}(\omega)$ was not function of the frequency, it was proposed to identify the unknown permutation matrix, by shuffling the column of $\hat{\mathbf{H}}(\omega)$, then computing $\tilde{\mathbf{H}}(\omega)$ and looking for the $\tilde{\mathbf{H}}(\omega)$ that contained coefficients with zeros only (no poles). Such an approach, however, is rather difficult to implement, and obviously does not apply to the case of frequency dependent permutation.

For the 2×2 case, it was proposed in [5] to extract two invariances from $\tilde{\mathbf{H}}(\omega) = \{\tilde{H}_{ij}(\omega)\}$, i.e.,

$$I_1^2(\omega) = \tilde{H}_{12}(\omega)/\tilde{H}_{21}(\omega) = H_{12}(\omega)/H_{21}(\omega) \quad (11)$$

$$I_2^2(\omega) = \tilde{H}_{21}(\omega) + 1/\tilde{H}_{12}(\omega) = H_{21}(\omega) + 1/H_{12}(\omega) \quad (12)$$

which, under assumptions (A5) and (A6) can lead to the cross-channels uniquely. In the next two sections we extend the ideas of [5] to general $n \times n$ case and show that it is possible to define invariances which suffice for the reconstruction of all mixing channels.

3. SOME DEFINITIONS

For each pair of rows $(i_1, i_2),\ i_1, i_2 = 1, ..., n$ let us define the invariances:

$$I_1^n(\omega; i_1, i_2) \triangleq \sum_{j=1}^{n} \frac{\tilde{H}_{i_1 j}(\omega)}{\tilde{H}_{i_2 j}(\omega)} = \sum_{j=1}^{n} \frac{H_{i_1 j}(\omega)}{H_{i_2 j}(\omega)}$$

$$I_2^n(\omega; i_1, i_2) \triangleq \sum_{j_1=1}^{n} \sum_{j_2>j_1}^{n} \frac{\tilde{H}_{i_1 j_1}(\omega)}{\tilde{H}_{i_2 j_1}(\omega)} \frac{\tilde{H}_{i_1 j_2}(\omega)}{\tilde{H}_{i_2 j_2}(\omega)} = \sum_{j_1=1}^{n} \sum_{j_2>j_1}^{n} \frac{H_{i_1 j_1}(\omega)}{H_{i_2 j_1}(\omega)} \frac{H_{i_1 j_2}(\omega)}{H_{i_2 j_2}(\omega)}$$

$$I_3^n(\omega; i_1, i_2) \triangleq \sum_{j_1=1}^{n} \sum_{j_2>j_1}^{n} \sum_{j_3>j_2}^{n} \frac{\tilde{H}_{i_1 j_1}(\omega)}{\tilde{H}_{i_2 j_1}(\omega)} \frac{\tilde{H}_{i_1 j_2}(\omega)}{\tilde{H}_{i_2 j_2}(\omega)} \frac{\tilde{H}_{i_1 j_3}(\omega)}{\tilde{H}_{i_2 j_3}(\omega)} = \sum_{j_1=1}^{n} \sum_{j_2>j_1}^{n} \sum_{j_3>j_2}^{n} \frac{H_{i_1 j_1}(\omega)}{H_{i_2 j_1}(\omega)} \frac{H_{i_1 j_2}(\omega)}{H_{i_2 j_2}(\omega)} \frac{H_{i_1 j_3}(\omega)}{H_{i_2 j_3}(\omega)}$$

$$\vdots$$

$$I_n^n(\omega; i_1, i_2) \triangleq \prod_{j=1}^{n} \frac{\tilde{H}_{i_1 j}(\omega)}{\tilde{H}_{i_2 j}(\omega)} = \prod_{j=1}^{n} \frac{H_{i_1 j}(\omega)}{H_{i_2 j}(\omega)}$$

It is easy to verify that these quantities are the same, whether they are defined based on $\mathbf{H}(\omega)$ or $\tilde{\mathbf{H}}(\omega)$.

Let us also define

$$P^n(\omega; i_1, i_2) = \prod_{j=1}^{n} H_{i_1 j}(\omega) H^*_{i_2 j}(\omega) \quad (13)$$

The phase of $P^n(\omega;i_1,i_2)$ equals the phase of $I_n^n(i_1,i_2)$ within a linear phase component. The time domain equivalent of $P^n(\omega;i_1,i_2)$ is $h_{i_1 1}(k)*\cdots*h_{i_1 n}(k)*h_{i_2 1}(-k)*\cdots*h_{i_2 n}(-k)$, where "*" denotes convolution. It is well established that a FIR sequence that doesn't contain zero-phase convolutional components can be reconstructed within a scalar from its phase only, even if the phase is known within a linear phase component [9]. Thus, under assumptions (A5) and (A6), $P^n(i_1,i_2)$ can be computed within a scalar constant, i.e, $c(i_1,i_2)^2$. Based on the computed $P^n(\omega;i_1,i_2)$ and the invariance $I_n^n(\omega;i_1,i_2)$ we define:

$$\begin{aligned} M_{i1}(\omega;i_1,i_2) &\triangleq |P^n(\omega;i_1,i_2)I_n^n(\omega;i_1,i_2)|^{1/2} \\ &= c(i_1,i_2)\prod_{j=1}^{n}|H_{i_1 j}(\omega)| \end{aligned} \quad (14)$$

After these definitions we are ready to proceed with the algorithm for resolving the permutation ambiguity.

4. RESOLVING THE FREQUENCY-DEPENDENT PERMUTATION AMBIGUITY

Proposition 2: *Under the assumptions (A1)-(A6), the channel matrix $\mathbf{H}(l)$ of an $n\times n$ MIMO system can be reconstructed exactly based on the invariances $I_k^n(\omega;i_1,i_2)$, $k,i_1,i_2=1,\cdots,n$.*

Proof: Since in general $n\times n$ case the notation becomes difficult to follow, we first present the 3×3 case. We start with the following invariances:

$$\begin{aligned}
I_1^3(\omega;1,2) &= \frac{1}{H_{21}(\omega)} + H_{12}(\omega) + \frac{H_{13}(\omega)}{H_{23}(\omega)} \\
I_2^3(\omega;1,2) &= \frac{H_{12}(\omega)}{H_{21}(\omega)} + \frac{H_{13}(\omega)}{H_{21}(\omega)H_{23}(\omega)} + \frac{H_{12}(\omega)H_{13}(\omega)}{H_{23}(\omega)} \\
I_3^3(\omega;1,2) &= \frac{\tilde{H}_{12}(\omega)\tilde{H}_{13}(\omega)}{\tilde{H}_{21}(\omega)\tilde{H}_{23}(\omega)} = \frac{H_{12}(\omega)H_{13}(\omega)}{H_{21}(\omega)H_{23}(\omega)} \\
I_1^3(\omega;1,3) &= \frac{1}{H_{31}(\omega)} + \frac{H_{12}(\omega)}{H_{32}(\omega)} + H_{13}(\omega) \\
I_2^3(\omega;1,3) &= \frac{H_{12}(\omega)}{H_{31}(\omega)H_{32}(\omega)} + \frac{H_{13}(\omega)}{H_{31}(\omega)} + \frac{H_{12}(\omega)H_{13}(\omega)}{H_{32}(\omega)} \\
I_3^3(\omega;1,3) &= \frac{H_{12}(\omega)H_{13}(\omega)}{H_{31}(\omega)H_{32}(\omega)} \\
I_1^3(\omega;2,3) &= \frac{1}{H_{32}(\omega)} + H_{23}(\omega) + \frac{H_{21}(\omega)}{H_{31}(\omega)} \\
I_2^3(\omega;2,3) &= \frac{H_{23}(\omega)}{H_{32}(\omega)} + \frac{H_{21}(\omega)}{H_{31}(\omega)H_{32}(\omega)} + \frac{H_{21}(\omega)H_{23}(\omega)}{H_{31}(\omega)} \\
I_3^3(\omega;2,3) &= \frac{H_{21}(\omega)H_{23}(\omega)}{H_{31}(\omega)H_{32}(\omega)}
\end{aligned}$$

Let us consider the polynomials:

$$x^3 - I_1^3(\omega;1,2)x^2 + I_2^3(\omega;1,2)x - I_3^3(\omega;1,2) = 0 \quad (15)$$

$$x^3 - I_1^3(\omega;1,3)x^2 + I_2^3(\omega;1,3)x - I_3^3(\omega;1,3) = 0 \quad (16)$$

$$x^3 - I_1^3(\omega;2,3)x^2 + I_2^3(\omega;2,3)x - I_3^3(\omega;2,3) = 0 \quad (17)$$

The roots of the first, second and third polynomial, respectively, are:

$$(X_1(\omega),X_2(\omega),X_3(\omega)) \to (\frac{1}{H_{21}(\omega)}, H_{12}(\omega), \frac{H_{13}(\omega)}{H_{23}(\omega)}) \quad (18)$$

$$(Y_1(\omega),Y_2(\omega),Y_3(\omega)) \to (\frac{1}{H_{31}(\omega)}, H_{13}(\omega), \frac{H_{12}(\omega)}{H_{32}(\omega)}) \quad (19)$$

$$(Z_1(\omega),Z_2(\omega),Z_3(\omega)) \to (\frac{1}{H_{32}(\omega)}, H_{23}(\omega), \frac{H_{21}(\omega)}{H_{31}(\omega)}) \quad (20)$$

Under assumptions (A5) and (A6), $P^3(\omega;1,2)$, $P^3(\omega;2,3)$ and $P^3(\omega;3,1)$ can be reconstructed within the scalars, $c(1,2)^2$, $c(2,3)^2$ and $c(3,1)^2$, respectively, thus we can obtain:

$$\begin{aligned} M_1(\omega;1,2) &= |P^3(\omega;1,2)I_3^3(\omega;1,2)|^{1/2} \\ &= c(1,2)|H_{12}(\omega)||H_{13}(\omega)| \end{aligned} \quad (21)$$

$$\begin{aligned} M_2(\omega;2,3) &= |P^3(\omega;2,3)I_3^3(\omega;2,3)|^{1/2} \\ &= c(2,3)|H_{21}(\omega)||H_{23}(\omega)| \end{aligned} \quad (22)$$

$$\begin{aligned} M_3(\omega;3,1) &= |P^3(\omega;3,1)I_3^3(\omega;3,1)|^{1/2} \\ &= c(3,1)|H_{31}(\omega)||H_{32}(\omega)| \end{aligned} \quad (23)$$

Let us consider a set of N frequencies in $[0,2\pi)$, where $N>2L$, i.e., the frequencies $\omega_m=\frac{2\pi}{N}m$, $m=0,1,...N-1$. Let us determine the roots $X_1(\omega_m)$, $X_2(\omega_m)$, $X_3(\omega_m)$, $Y_1(\omega_m)$, $Y_2(\omega_m)$, $Y_3(\omega_m)$, $Z_1(\omega_m)$, $Z_2(\omega_m)$ and $Z_3(\omega_m)$ for all ω_m, $m=0,1,...N-1$, and then form the following sets of coefficients:

$$\begin{aligned} K(\omega_m;1,2) &\triangleq \{k_{ij}^{(1)}(\omega_m;1,2), i,j=1,2,3\}, \\ k_{ij}(\omega_m;1,2) &= \frac{M_1(\omega_m;1,2)}{|X_i(\omega_m)||Y_j(\omega_m)|} \end{aligned} \quad (24)$$

$$\begin{aligned} K(\omega_m;2,3) &\triangleq \{k_{ij}(\omega_m;2,3), i,j=1,2,3\}, \\ k_{ij}(\omega_m;2,3) &= \frac{M_2(\omega_m;2,3)|X_i(\omega_m)|}{|Z_j(\omega_m)|} \end{aligned} \quad (25)$$

$$\begin{aligned} K(\omega_m;3,1) &\triangleq \{k_{ij}(\omega_m;3,1), i,j=1,2,3\}, \\ k_{ij}(\omega_m;3,1) &= M_3(\omega_m;3,1)|Y_i(\omega_m)||Z_j(\omega_m)| \end{aligned} \quad (26)$$

By letting $X_i(\omega_m)$ and $Y_j(\omega_m)$ take all possible values allowed by (18) and (19), and based on the form of $M_1(\omega;1,2)$, it can easily be verified that for each frequency ω_m there will be one and only one combination that will yield an element independent of frequency and equal to $c(1,2)$, i.e.,

$$K(\omega_0;1,2)\cap K(\omega_1;1,2)\cap ...\cap K(\omega_{N-1};1,2) = \{c(1,2)\} \quad (27)$$

Similarly, comparing (18), (20) and (22), and (19), (20) and (23) we conclude:

$$K(\omega_0;2,3)\cap K(\omega_1;2,3)\cap ...\cap K(\omega_{N-1};2,3) = \{c(2,3)\} \quad (28)$$

$$K(\omega_0;2,3)\cap K(\omega_1;2,3)\cap ...\cap K(\omega_{N-1};2,3) = \{c(3,1)\} \quad (29)$$

It can easily be verified that the last three equations can be violated only if one of the following is true:

$$\begin{aligned} |H_{ij}(\omega)||H_{ji}(\omega)| &= const. \\ |H_{ij}(\omega)||H_{jk}(\omega)||H_{ki}(\omega)| &= const., \quad i \neq j \neq k \\ \frac{|H_{ij}(\omega)||H_{jk}(\omega)|}{|H_{ik}(\omega)|} &= const., \quad i \neq j \neq k \\ |H_{ij}(\omega)||H_{ji}(\omega)||H_{ik}(\omega)||H_{ki}(\omega)| &= const., \quad i \neq j \neq k \end{aligned}$$

However, because of assumptions (A5) and (A6) none of the above can happen.

Based on the obtained constants $c(1,2)$, $c(2,3)$ and $c(3,1)$ (see Eqs. (27)-(29)) we now can recover all cross-channels using the following relations [1]:

$$k_{ij}(\omega_m; 1,2) = c(1,2) \quad \Rightarrow \quad H_{12}(\omega_m) = X_i(\omega_m), \quad H_{13}(\omega_m) = Y_j(\omega_m) \tag{30}$$

$$k_{ij}(\omega_m; 2,3) = c(2,3) \quad \Rightarrow \quad H_{21}(\omega_m) = \frac{1}{X_i(\omega_m)}, \quad H_{23}(\omega_m) = Z_j(\omega_m) \tag{31}$$

$$k_{ij}(\omega_m; 3,1) = c(3,1) \quad \Rightarrow \quad H_{31}(\omega_m) = \frac{1}{Y_i(\omega_m)}, \quad H_{32}(\omega_m) = \frac{1}{Z_j(\omega_m)} \tag{32}$$

Let us now consider the general $n \times n$ case. Let us assume that we want to reconstruct an arbitrary channel $h_{i_1 i_2}$, where $i_1, i_2 \in \{1, 2, ..., n\}$. We start with the following quantity that we are able to obtain as previously discussed:

$$\begin{aligned} M_{i1}(\omega; i_1, r) &\stackrel{\triangle}{=} |P^n(\omega; i_1, r) I_n^n(\omega; i_1, r)|^{1/2} \\ &= c(i_1, r) \prod_{j=1}^{n} |H_{i_1 j}(\omega)|, \quad r \in \{1, ..., n\} \end{aligned} \tag{33}$$

Let us consider the following polynomial:

$$x^n - I_1^n(\omega; i_1, p) x^{n-1} + I_2^n(\omega; i_1, p) x^{n-2} - ... + (-1)^n I_n^n(\omega; i_1, p) \quad , \; p \in \{1, 2, ..., n\} \tag{34}$$

and let $X_i^p(\omega)$, $i = 1, 2, ...n$ denotes its i-th root. Then it is easy to show that $X_i^p(\omega)$ will be equal to one of the following:

$$\frac{H_{i_1 1}(\omega)}{H_{p1}(\omega)}, \quad \frac{H_{i_1 2}(\omega)}{H_{p2}(\omega)}, \quad . \quad . \quad . \quad , \quad \frac{H_{i_1 n}(\omega)}{H_{pn}(\omega)} \tag{35}$$

For the previously selected r, let us define the following set:

$$K(\omega_m; i_1, r) = \{k_{j_1 j_2 \dots j_n}(\omega_m; i_1, r)\}, \qquad m = 0, 1, ..., N-1 \tag{36}$$

where $\omega = \frac{2\pi}{N}\omega_m$, $m = 0, ..., N-1$, and

$$\begin{aligned} k_{j_1 j_2 \dots j_n}(\omega_m; i_1, r) &= \\ &= \frac{M_{i_1}(\omega_m; i_1, r)}{|X_{j_1}^1(\omega_m)||X_{j_2}^2(\omega_m)|...|X_{j_p}^p(\omega_m)|...|X_{j_n}^n(\omega_m)|}, \\ & \qquad j_1, j_2, ...j_n = 1, 2, ..., n \end{aligned} \tag{37}$$

[1]Note that it is possible that for certain frequencies $\tilde{\omega}_m$, set $k_{ij}(\tilde{\omega}_m; i_1, i_2)$ has two or more elements that equal $c(i_1, i_2)$. At these frequencies, the ambiguity cannot be resolved. However, under assumptions (A5) and (A6), we can always make N large enough to have sufficient number of frequencies without ambiguity for channel reconstruction.

According to (35), and under assumptions (A5) and (A6), one and only one element of this set will not depend on the frequency, corresponding to:

$$\begin{aligned} X_{j_1}^1(\omega_m) &= \frac{|H_{i_1 1}(\omega_m)|}{|H_{11}(\omega_m)|} = |H_{i_1 1}(\omega_m)| \\ X_{j_2}^2(\omega_m) &= \frac{|H_{i_1 2}(\omega_m)|}{|H_{22}(\omega_m)|} = |H_{i_1 2}(\omega_m)| \\ &\vdots \\ X_{j_n}^n(\omega_m) &= \frac{|H_{i_1 n}(\omega_m)|}{|H_{nn}(\omega_m)|} = |H_{i_1 n}(\omega_m)| \end{aligned}$$

Therefore, for N large enough we have:

$$K(\omega_0; i_1, r) \cap K(\omega_1; i_1, r) \cap ... \cap K(\omega_{N-1}; i_1, r) = \{c(i_1, r)\} \tag{38}$$

After determining the scalar constant $c(i_1, r)$, assuming no ambiguity at that certain frequency), $H_{i_1 i_2}(\omega_m)$ can be obtained as:

$$k_{j_1 j_2 \dots j_n}(\omega_m; i_1, r) = c(i_1, r) \quad \Rightarrow \quad H_{i_1 i_2}(\omega_m) = X_{j_{i_2}}^{i_2}(\omega_m) \tag{39}$$

Therefore, we were able to recover an arbitrary cross-channel $h_{i_1 i_2}(k)$, $i_1, i_2 \in \{1, 2, ..., n\}$ thus Proposition 2 is proved.

5. CONCLUSIONS

We consider a general $n \times n$ MIMO system excited by unobservable inputs that are spatially independent, cyclostationary with unknown statistics. Via propositions 1 and 2, and under assumptions (A1)-(A6) we showed that the system is uniquely identifiable based on frequency domain second-order correlations of the system output. Although the proof is constructive, further work is needed if one would wish to turn this identifiability result to a system estimation method.

6. REFERENCES

[1] A. Belouchrani, K. Abed-Meraim, J.-F. Cardoso, and E. Moulines, "A Blind Source Separation Technique Using Second-Order Statistics," *IEEE Trans. on Signal Processing*, vol. 45(2), pp. 434-444, February 1997.

[2] J.-F. Cardoso and A. Souloumiac, "Blind Beamforming for Non-Gaussian Signals," *Proc. IEE, pt. F*, vol. 140(6), pp. 362-370, December 1993.

[3] J.-F. Cardoso, "Blind Signal Separation: Statistical Principles," *Proc. IEEE*, vol. 86(10), pp. 2009-2025, October 1998.

[4] B. Chen and A.P. Petropulu, "Frequency Domain Blind MIMO System Identification Based on Second and Higher-Order Statistics," *IEEE Trans. on Signal Processing*, vol. 49(8), pp. 1677-1688, August 2001.

[5] K.I. Diamantaras, A.P. Petropulu and B. Chen, "Blind Two-Input-Two-Output FIR Channel Identification Based on Frequency Domain Second-Order Statistics," *IEEE Trans. on Signal Processing*, vol. 48(2), pp. 534-542, February 2000.

[6] Y. Inouye and K. Hirano, "Cumulant-Based Blind Identification of Linear Multi-Input-Multi-Output Systems Driven by Colored Inputs", *IEEE Trans. on Signal Processing*, vol. 45 (6), pp. 1543–1552, June 1997.

[7] U.A. Lindgren and H. Broman, "Source separation using a criterion based on second-order statistics,"*IEEE Trans. on Signal Processing*, vol. 46(7), pp. 1837–1850, July 1998.

[8] L. Parra and C. Spence, "Convolutive Blind Separation of Non-Stationary Sources," *IEEE Tr. Speech and Audio Processing*, Vol. 8, No. 3, pp.320-327, May 2000.

[9] H. Pozidis and A.P. Petropulu, "Cross-spectrum based blind channel estimation," *IEEE Trans. on Signal Processing*, vol. 45 (12), pp. 2977-2993, December 1997.

[10] S. Shamsunder and G.B. Giannakis, "Multichannel blind signal separation and reconstruction," *IEEE Trans. on Speech & Audio Processing*, vol. 5(6), pp. 515-527, November 1997.

[11] L. Tong, R. Liu, V.C. Soon, and Y. Huang, "Indterminacy and Identifiability of Blind Identification," *IEEE Trans. on Circuits and Systems*, vol. 38(5), pp. 499-509, May 1991.

[12] D. Yellin and E. Weinstein, "Multichannel signal separation: methods and analysis," *IEEE Trans. on Signal Processing*, vol. 44, pp. 106-118, Jan. 1996.